2

SAINT PETERSBURG STATE UNIVERSITY STUDIES IN MATHEMATICS

Marat V. Yuldashev

Nonlinear Mathematical Models of Costas Loops

Saint Petersburg | 2013

The series *Saint Petersburg State University Studies in Mathematics* presents final results of research carried out in postgraduate mathematics programs at St. Petersburg State University. Most of this research is here presented after publication in leading scientific journals.

The supervisors of these works are well-known scholars of St. Petersburg State University and invited foreign researchers. The material of each book has been considered by a permanent editorial board as well as a special international commission comprised of well-known Russian and international experts in their respective fields of study.

Printed in Russia by St. Petersburg University Press
11/21 6th Line, St. Petersburg, 199004

ISBN 978-5-288-05425-9
ISSN 2308-3476

ABSTRACT

Yuldashev, Marat V.
Nonlinear Mathematical Models of
Costas Loops
Saint Petersburg: Saint Petersburg State University, 2013, 40 p.(+included articles)
Saint Petersburg State University Studies in Mathematics, Vol. 2
ISBN 978-5-288-05425-9, ISSN **2308-3476**

This work is devoted to the development of nonlinear mathematical models of Costas loops. A Costas loop was invented in 1956 by John P. Costas of General Electric. Nowadays, a Costas loop is widely used in many applications including telecommunication devices, global positioning systems (GPS, GLONASS), medical implants, mobile phones, and other gadgets.

In contrast to the phase-locked loop (PLL) based circuit, the Costas loop is designed to simultaneously perform two tasks — carrier recovery and data demodulation. The direct application of a PLL to these tasks is possible, but it is not effective, because after superimposing the transmitted data and carrier signal, frequent changes of transmitted data require that a PLL constantly adjusts itself. A Costas loop is designed in such a way that the transmitted data doesn't affect transient processes and does not require frequent tuning. The requirement for simultaneous data demodulation and carrier recovery makes the Costas loop-based devices multi-loop, multi-channel circuits with multiple outputs. Also, in contrast to a PLL, the Costas loop has three non-linear elements. All this makes the development of non-linear models of Costas loops a difficult task. High-frequency signals, used in the modern devices, further complicate the application of analytical methods and numerical simulation. This is due to the fact that the transient time is greater than the signal's periods by several orders of magnitude. Furthermore, the behaviour of Costas circuits greatly depends on the classes of signals involved. So, the development of non-linear mathematical models of Costas loops that allow one to facilitate the application of analytical methods and reduce the numerical simulation time is a relevant problem of the practical significance. It is this problem that is considered and solved in the present study.

In this work, nonlinear mathematical models of the classic Costas loop and the Quadrature Phase Shift Keying (QPSK) Costas loop have been developed. All theoretical results are rigorously proved. An effective numerical procedure for the simulation of Costas loops based on the phase-detector characteristics is proposed.

The results of the study have been published in 22 papers (8 of which are indexed in Scopus).

Keywords: Costas loop, carrier tracking,GPS, GLONASS, PLL, BPSK, QPSK Costas

Supervisors Dr. Nikolay V. Kuznetsov
Department of Applied Cybernetics
Faculty of Mathematics and Mechanics
Saint Petersburg State University, Russia,
Faculty of Information Technology
University of Jyväskylä, Finland

Professor Gennady A. Leonov
Member (corr.) of Russian Academy of Science,
Head of Department of Applied Cybernetics,
Dean of Faculty of Mathematics and Mechanics
Saint Petersburg State University, Russia

Professor Pekka Neittaanmäki
Department of Mathematical Information Technology,
Dean of Faculty of Information Technology
University of Jyväskylä, Finland,
Honorary Doctor of Saint Petersburg State University, Russia

Opponents Professor Alexey S. Matveev (Chairman)
Faculty of Mathematics and Mechanics
St. Petersburg State University, Russia,
Electrical & Electronic Engineering
and Telecommunications School
University of New South Wales, Australia

Professor Boris R. Andrievsky
Faculty of Mathematics and Mechanics
St. Petersburg State University, Russia,
Faculty of Information and Control Systems
Baltic State Technical University "VOENMEH", Russia

Professor Alexander K. Belyaev
Director of Institute of Applied Mathematics & Mechanics
St. Petersburg State Polytechnical University, Russia,
Vice-Director of Institute for Problems in Mechanical
Engineering Russian Academy of Sciences, Russia,
Honorary Doctor of University of Johannes Kepler, Austria

Professor Vladimir I. Nekorkin
Faculty of Radiophysics,
Lobachevsky State University of Nizhni Novgorod, Russia,
Head of Department of Nonlinear Dynamics
Institute of Applied Physics
Russian Academy of Sciences, Russia

Professor Sergei Yu. Pilyugin
Faculty of Mathematics and Mechanics
St. Petersburg State University, Russia

Professor Vladimir Rasvan
Faculty of Automatics, Computers and Electronics,
Director of Research Center
"Nonlinear control. Stability and oscillations"
University of Craiova, Romania

Professor Timo Tiihonen
Department of Mathematical Information Technology,
Vice-Dean of Faculty of Information Technology,
University of Jyväskylä, Finland

ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to my supervisors Dr. Nikolay V. Kuznetsov, Prof. Gennady A. Leonov, and Prof. Pekka Neittaanmäki for their guidance and continuous support.

I greatly appreciate the opportunity to participate in Educational and Research Double Degree Programme organized by the Department of Applied Cybernetics (Saint Petersburg State University) and the Department of Mathematical Information Technology (University of Jyväskylä).

This work was funded by the grants from Saint Petersburg State University (Russia), Federal Target Programme of Ministry of Education and Science (Russia), and Scholarship of the President of Russia.

I'm very grateful to Prof. Sergei Abramovich (The State University of New York at Potsdam, USA) for his valuable comments.

I would like to extend my deepest thanks to my parents Prof. Dilara Kalimullina and Prof. Vladimir Yuldashev.

LIST OF FIGURES

CONTENTS

LIST OF INCLUDED ARTICLES

PI G.A. Leonov, N.V. Kuznetsov, M.V. Yuldashev, M.V. Yuldashev. Differential Equations of Costas Loop. *Doklady Mathematics, Vol. 86, No. 2, pp. 723–728*, 2012 [Scopus].

PII N.V. Kuznetsov, G.A. Leonov, M.V. Yuldashev, M.V. Yuldashev. Nonlinear Analysis of Costas Loop Circuit. *ICINCO 2012 - Proceedings of the 9th International Conference on Informatics in Control, Automation and Robotics, Vol. 1, pp. 557–560*, 2012 [Scopus].

PIII N.V. Kuznetsov, G.A. Leonov, P. Neittaanmäki, M.V. Yuldashev, M.V. Yuldashev. Nonlinear Mathematical Models of Costas Loop for General Waveform of Input Signal. *IEEE 4th International Conference on Nonlinear Science and Complexity, NSC 2012 - Proceedings, pp. 75–80*, 2012 [Scopus].

PIV N.V. Kuznetsov, G.A. Leonov, P. Neittaanmäki, S.M. Seledzhi, M.V. Yuldashev, M.V. Yuldashev. Nonlinear Analysis of Costas Loop Circuit. *ICINCO 2013 - Proceedings of the 9th International Conference on Informatics in Control, Automation and Robotics, [accepted]*, 2013 [Scopus].

PV R.E. Best, N.V. Kuznetsov, G.A. Leonov, M.V. Yuldashev, R.V. Yuldashev. Nonlinear Analysis of Phase-Locked Loop Based Circuits. *Discontinuity and Complexity in Nonlinear Physical Systems (eds. J.T. Machado, D. Baleanu, A. Luo), Springer [accepted]*, 2013.

OTHER PUBLICATIONS

AI N. V. Kuznetsov, G. A. Leonov, S.M. Seledzhi, M. V. Yuldashev, R. V. Yuldashev. Nonlinear Analysis of Phase-Locked Loop with Squarer. *IFAC Proceedings Volumes (IFAC-PapersOnline) (5th IFAC International Workshop on Periodic Control Systems, Caen, France) [accepted] 2013, [Scopus]*

AII N. V. Kuznetsov, G. A. Leonov, M. V. Yuldashev, R. V. Yuldashev. Phase Synchronization in Analog and Digital Circuits. *Plenary lecture, SPb:5-aya rossijskaya Mul'tiKonferentsiya po Problemam Upravleniya*, pp. 24–31, 2012.

AIII N. V. Kuznetsov, G. A. Leonov, M. V. Yuldashev, R. V. Yuldashev. Nonlinear Analysis of Analog Phase-Locked Loop. *Proceedings of International conference Dynamical Systems and Applications*, pp. 21–22, 2012.

AIV G. A. Leonov, N. V. Kuznetsov, M. V. Yuldashev, R. V. Yuldashev. Analytical Method for Computation of Phase-Detector Characteristic. *IEEE Transactions on Circuits and Systems II: Express briefs*, Vol. 59, Iss. 10, pp. 633–637, 2012 [Scopus].

AV N. V. Kuznetsov, G. A. Leonov, P. Neittaanmäki, S. M. Seledzhi, M. V. Yuldashev, R. V. Yuldashev. Simulation of Phase-Locked Loops in Phase-Frequency Domain. *International Congress on Ultra Modern Telecommunications and Control Systems and Workshops*, pp. 351–356, 2012 [Scopus].

AVI M.V. Yuldashev. Nonlinear analysis of Costas loop. *XII International Workshop "Stability and Oscillations of Nonlinear Control Systems"*, pp. 348–350, 2012.

AVII M.V. Yuldashev. Nonlinear Analysis of BPSK Device. *Proceedings of the 3rd Inter-University Conference on Informatics*, pp. 457–458, 2012 [in Russian].

AVIII G. A. Leonov, N. V. Kuznetsov, M. V. Yuldashev, R. V. Yuldashev. Computation of Phase Detector Characteristics in Synchronization Systems. *Doklady Mathematics*, Vol. 84, No. 1, pp. 586–590, 2011 [Scopus].

AIX N. V. Kuznetsov, G. A. Leonov, M. V. Yuldashev, R. V. Yuldashev. Analytical methods for computation of phase-detector characteristics and PLL design. *International Symposium on Signals, Circuits and Systems*, pp. 1–4, IEEE press, 2011 [Scopus].

AX N. V. Kuznetsov, G. A. Leonov, P. Neittaanmäki, S. M. Seledzhi, M. V. Yuldashev, R. V. Yuldashev. High-frequency analysis of phase-locked loop and phase detector characteristic computation. *8th International Conference on Informatics in Control, Automation and Robotics (ICINCO 2011)*, Vol. 1, pp. 272–278, INSTICC Press, 2011 [Scopus].

AXI M.V. Yuldashev. Phase Detector Characteristics Computation for Two Impulse Signals. *Proceedings of the 2nd Inter-University Conference on Informatics*, pp. 389–390, 2011 [in Russian].

AXII N. V. Kuznetsov, G. A. Leonov, P. Neittaanmäki, S. M. Seledzhi, M. V. Yuldashev, R. V. Yuldashev. Nonlinear Analysis of Phase-locked loop. *IFAC Proceedings Volumes (IFAC-PapersOnline)*, Vol. 4, No. 1, pp. 34–38, 2010.

PATENTS

AXIII N. V. Kuznetsov, G. A. Leonov, P. Neittaanmäki, M. V. Yuldashev, R. V. Yuldashev. *Patent application.* Method And System For Modeling Costas Loop Feedback For Fast Mixed Signals Solutions. FI 20130124, 2013.

AXIV N. V. Kuznetsov, G. A. Leonov, S. M. Seledzhi, M. V. Yuldashev, R. V. Yuldashev. Patent. Modulator of Phase Detector Parameters. RU 2449463 C1, 2011.

AXV N. V. Kuznetsov, G. A. Leonov, S. M. Seledzhi, M. V. Yuldashev, R. V. Yuldashev. Patent. The Method and Device for Determining of Characteristics of Phase-Locked Loop Generators. Sposob dlya opredeleniya rabochikh parametrov fazovoj avtopodstrojki chastoty generatora i ustrojstvo dlya ego realizatsii. RU 11255 U1, 2011.

AXVI N. V. Kuznetsov, G. A. Leonov, S. M. Seledzhi, M. V. Yuldashev, R. V. Yuldashev. Avtorskoe svidetelstvo na programmu. Program for Determining and Simulation of the Main Characteristics of Phase-Locked Loops (MR). RU 2011613388, 2011.

AXVII N. V. Kuznetsov, G. A. Leonov, S. M. Seledzhi, M. V. Yuldashev, R. V. Yuldashev. Avtorskoe svidetelstvo na programmu. Program for Determining and Simulation of the Main Characteristics of Costas Loops (CLMod). RU 2011616770, 2011.

1 INTRODUCTION

1.1 Intellectual merit

The classic Costas loop was invented in 1956 by famous American electrical engineer John P. Costas of General Electric (Costas, 1956, 1962). A Costas loop is a carrier tracking and Binary Phase Shift Keying (BPSK) data demodulation device (Tomasi, 2001; Young, 2004; Couch, 2007; Proakis and Salehi, 2007; Best, 2007; Chen et al., 2010). Nowadays, a Costas loop and its modifications are widely used in telecommunication devices (Viterbi, 1983; Malyon, 1984; Hodgkinson, 1986; Stephens, 2001; Yu et al., 2011; Abe et al., 2012), Global Positioning Systems (GPS) (Jasper, 1987; Beier, 1987; Mileant and Hinedi, 1994; Braasch and Van Dierendonck, 1999; Tanaka et al., 2001; Humphreys et al., 2005; An'an and Du Yong, 2006; Kaplan and Hegarty, 2006; Tang et al., 2010; Misra and Palod, 2011; Hegarty, 2012), medical implants (Hu and Sawan, 2005; Luo and Sonkusale, 2008; Xu et al., 2009), mobile phones (Kobayashi et al., 1992; Gao and Feher, 1996; Lin et al., 2004; Shah and Sinha, 2009), and other important technological applications (Wang and Leeb, 1987; Miyazaki et al., 1991; Djordjevic et al., 1998; Djordjevic and Stefanovic, 1999; Cho, 2006; Hayami et al., 2008; Nowsheen et al., 2010).

A mathematical description and the investigation of mathematical models of Costas loops is a very difficult task (Abramovitch, 2002). The direct description of these circuits leads to the analysis of nonlinear non-autonomous differential equations with high-frequency and low-frequency components in the right-hand sides of the equations (Leonov and Seledzhi, 2002; Leonov, 2006; Kudrewicz and Wasowicz, 2007; Leonov et al., 2009; Leonov, 2010). Because in the modern applications not only sinusoidal but many other classes of signal have been used (Henning, 1981; Wang and Emura, 1998; Sutterlin and Downey, 1999; Wang and Emura, 2001; Chang and Chen, 2008; Sarkar and Sengupta, 2010), it further complicates the study of the corresponding differential equations. However, it is possible to overcome these difficulties through the development of the high-frequency asymptotic analysis methods (see (Leonov, 2008; Kuznetsov et al., 2010a) and [PI—PV]). These methods allow one to "split" high-frequency and low-frequency parts in the mathematical models of Costas loops.

According to one of the largest publication database (www.sciencedirect.com), there exist a high interest in the investigation of Costas loops:

- 2008 — 503 publications
- 2009 — 500 publications
- 2010 — 512 publications
- 2011 — 607 publications
- 2012 — 680 publications

This work further contributes to the body of knowledge about Costas loops and it is devoted to the development and analysis of their mathematical models using the high-frequency asymptotic analysis approach.

1.2 Goal of the work

The goals of this work include: a rigorous mathematical derivation of models of the classic Costas loop for general signal waveforms, a modification of the QPSK Costas loop and their numerical simulation.

1.3 Methods of investigation

Many methods of analysis of Costas loops are considered in various publications. However, the problems of the development of adequate nonlinear models and rigorous analysis of such models are still far from being resolved. However, a simple linear analysis can lead to incorrect conclusions and, thereby, it requires rigorous justification. Numerical simulation is not a trivial task also due to the high-frequency properties of the signals involved. Therefore the development of nonlinear mathematical models of Costas loops is a must for the analysis of such systems.

A Costas loop is a PLL-based circuit and, thereby, methods similar to those used in the context of investigation of any PLL may be adapted here. The first mathematical description of physical models was proposed by Gardner and Viterbi (Gardner, 1966; Viterbi, 1966). These authors described a mathematical model of the classic Costas loop in the signal space and, without a rigorous mathematical justification, proposed a mathematical model in the phase space. Although PLL-based circuits are essentially nonlinear control systems, in the modern engineering literature devoted to the analysis of PLL-based circuits (Lindsey, 1972; Lindsey and Simon, 1973; Djordjevic et al., 1998; Djordjevic and Stefanovic, 1999; Fiocchi et al., 1992; Miyazaki et al., 1991; Cho, 2006; Wang and Leeb, 1987; Wang and Emura, 2001, 1998; Hayami et al., 2008; Young et al., 1992; Gardner et al., 1993; Gardner, 1993; Fines and Aghvami, 1991; Margaris, 2004; Kroupa, 2003;

Razavi, 2003; Shu and Sanchez-Sinencio, 2005; Manassewitsch, 2005; Egan, 2000; Suarez and Quere, 2003; Tretter, 2007; Emura, 1982; Benarjee, 2006; Demir et al., 2000a; Stephens, 2001; Xanthopoulos et al., 2001; Demir et al., 2000b; Roberts et al., 2009; Kim et al., 2010; Tomkins et al., 2009; Proakis and Salehi, 2007), the main means of investigation include the use of simplified linear models, methods of linear analysis, empirical rules, and numerical simulation (see a plenary lecture of D. Abramovitch at the 2002 American Control Conference (Abramovitch, 2002)). While the analysis of linearized models of control systems may lead to incorrect conclusions[1], attempts to justify the reliability of conclusions, based on the application of such simplified approaches, are quite rare (see, e.g., (Suarez and Quere, 2003; Margaris, 2004; Banerjee and Sarkar, 2008a; Chicone and Heitzman, 2013; Best, 2007; Suarez et al., 2012; Feely, 2007; Feely et al., 2012; Kudrewicz and Wasowicz, 2007; Sarkar and Sengupta, 2010; Banerjee and Sarkar, 2008b)). A rigorous nonlinear analysis of a PLL-based circuit models is often a very difficult task (Leonov and Seledzhi, 2002; Kuznetsov, 2008; Kudrewicz and Wasowicz, 2007); therefore, for the analysis of nonlinear PLL models numerical simulations are widely used (Troedsson, 2009; Best, 2007; Bouaricha et al., 2012). However, for the high-frequency signals, complete numerical simulation of the physical model of a PLL-based circuit in signal/time space, described by nonlinear non-autonomous system of differential equations, is highly complicated since it is necessary to simultaneously observe "*very fast time scale of the input signals*" and "*slow time scale of signal's phases*" (Abramovitch, 2008a,b). Here, a relatively small discretization step in a numerical procedure does not allow one to consider transition processes for the high-frequency signals in a reasonable time period.

To overcome these difficulties, it was suggested (Gardner, 1966; Viterbi, 1966) to construct a dynamical model of a Costas loop circuit in the space of signal phases. As noted in [PIV], this approach considers only a slow time scale of the signals phases. This requires the computation of the phase detector (PD) characteristic, which depends on waveforms of circuit signals (Leonov, 2008; Kuznetsov et al., 2009b,a, 2008; Leonov et al., 2006, 2009; Kuznetsov, 2008). However, in order to use the results of such analysis of a mathematical model describing the behaviour of the corresponding physical model, a rigorous justification is needed [PI — PV]. To this end, it is essential, in turn, to apply also the averaging methods (Krylov and Bogolyubov, 1947; Mitropolsky and Bogolubov, 1961).

---

[1] see, e.g. counterexamples to the filter hypothesis and to Aizerman's and Kalman's conjectures regarding absolute stability (Kuznetsov and Leonov, 2001; Leonov et al., 2010c,b,a,b; Bragin et al., 2010; Leonov et al., 2011c; Leonov and Kuznetsov, 2011; Kuznetsov et al., 2011b; Bragin et al., 2011; Leonov et al., 2011b; Leonov and Kuznetsov, 2012, 2013b,c,d; Kuznetsov and Leonov, 2008; Kuznetsov et al., 2010b; Leonov et al., 2010c; Leonov and Kuznetsov, 2010; Leonov et al., 2011a; Kuznetsov et al., 2011a,c; Leonov et al., 2011d, 2012b,a; Kiseleva et al., 2012; Andrievsky et al., 2012; Kuznetsov et al., 2013a,b; Leonov and Kuznetsov, 2013a) and Perron effects of sign inversions of Lyapunov exponents for time varying linearizations (Kuznetsov and Leonov, 2005; Leonov and Kuznetsov, 2007) etc.

1.4 The main results

- Nonlinear mathematical models of the classic Costas loop for various signal waveforms (sinusoidal, impulse, polyharmonic, piecewise-differentiable) have been developed [see articles PI—PIV].
- A nonlinear mathematical model of the QPSK Costas loop is justified [see article PV].
- Effective numerical procedure and software tool for the simulation and analysis of Costas loops have been proposed [see articles PIII—PIV].

1.5 Adequacy of the results

All theoretical results are rigorously proved and, in special cases, coincide with the known classic results. Comparative numerical simulation of the proposed mathematical model and the corresponding physical model gave similar results.

1.6 Novelty

In this work, for the first time, a comprehensive rigorous mathematical method of constructing mathematical models of Costas loops for general signal waveforms is proposed. This method is based on the combination of engineering approaches to the investigation of PLL systems, the high-frequency analysis, and Fourier series application.

1.7 Practicability

The obtained results can be used for the analysis of the stability of Costas loops. The proposed method allows one to significantly reduce computation time spent on the numerical simulation of Costas loops (see patent application [AXVII]). It has become possible to obtain important characteristics of Costas circuits such as pull-in time, pull-in range, etc. Also, the models developed facilitate further analysis and synthesis of Costas loops.

1.8 Appraisal of the work and publications

The results of this work were presented at the following international conferences: International Congress on Ultra Modern Telecommunications and Control

Systems and Workshops (St.Petersburg, Russia – 2012), IEEE 4th International Conference on Nonlinear Science and Complexity (Budapest, Hungary – 2012), 9th International Conference on Informatics in Control, Automation and Robotics (Rome, Italy – 2012), International conference Dynamical Systems and Applications (Kiev, Ukrane – 2012), IEEE 10-th International Symposium on Signals, Circuits and Systems (Iasi, Romania – 2011), 8th International Conference on Informatics in Control, Automation and Robotics (Noordwijkerhout, The Netherlands – 2011), 4th IFAC Workshop on Periodic Control System (Antalya, Turkey – 2010), International Workshop "Mathematical and Numerical Modeling in Science and Technology" (Jyväskylä, Finland – 2010); at the seminars of the Department of Applied Cybernetics (St. Petersburg State University, Faculty of Mathematics and Mechanics) and at the seminars of the Department of Mathematical Information Technology (University of Jyväskylä, Finland).

The results of this work also appeared in 21 publications: 8 publications in Scopus database, one Finnish patent application, 2 Russian patents, 2 certificates for computer program. The main results of this work are included in (Yuldashev, 2012, 2013a,b).

In the included papers [PI–PV] co-authors formulated the problems and estimated integrals, the author formulated and proved theorems.

2 THE MAIN CONTENT

2.1 Nonlinear models of the classic Costas loop

Consider an operation of the classic Costas loop (see Fig. 1) with the sinusoidal carrier and sinusoidal VCO (Voltage-Controlled Oscillator) signals after transient processes. The input signal is a BPSK signal, which is a product of the transmit-

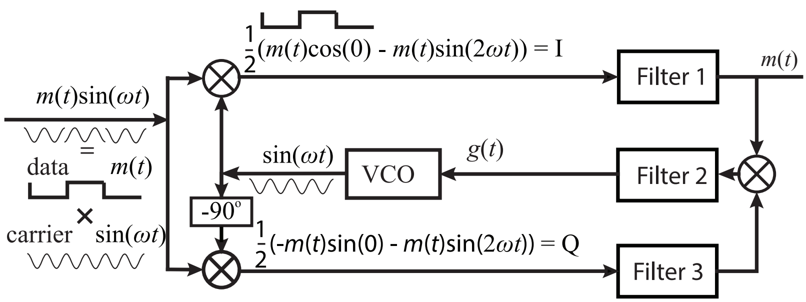


FIGURE 1 Costas loop in-lock state: $m(t)\sin(\omega t)$ is an input signal; $m(t) = (\pm 1)$ is the transmitted data; ω is the frequency of input carrier and VCO output

ted data $m(t) = \pm 1$ and the harmonic carrier $\sin(\omega t)$ with the high frequency ω. Because here the in-lock state of the Costas loop is considered, VCO signal is synchronized with the carrier (i.e., there is no frequency and phase difference between VCO signal and input carrier). On the lower branch (Q), after multiplying the VCO signal, shifted by $-\frac{\pi}{2}$, by the input signal one has

$$\begin{aligned} Q = \frac{1}{2}\big(m(t)\sin(0) - m(t)\sin(2\omega t)\big) = \\ = -\frac{1}{2}m(t)\sin(2\omega t). \end{aligned} \tag{1}$$

From an engineering point of view, the high-frequency part $\sin(2\omega t)$ in (1) is erased by a low-pass filter on Q the branch. Since $\sin(0) = 0$, the signal on the lower branch after the filtration is equal to zero, which indicates the in-lock state of the Costas loop.

On the upper branch (I), the input signal is multiplied by the output signal of VCO:

$$\begin{aligned} I &= \frac{1}{2}\big(m(t)\cos(0) - m(t)\cos(2\omega t)\big) = \\ &= \frac{1}{2}\big(m(t) + m(t)\cos(2\omega t)\big). \end{aligned} \tag{2}$$

The high frequency term $\cos(2\omega t)$ is filtered by the low-pass filter. Since $\cos(0) = 1$, on the upper branch, after filtration, one can obtain demodulated data $m(t)$. Then both branches are multiplied together and, after an additional low-pass filtration, one gets the signal $g(t)$ to adjust VCO frequency to the frequency of the input signal carrier. After the transient processes, the carrier and the VCO frequencies are equal to each other and the control input of VCO is equal to zero:

$$g(t) = 0.$$

These results, lacking rigorous mathematical justification, were well-known to engineers (Gardner, 1966; Viterbi, 1966) in the case of sinusoidal signals. The first effective mathematical model of high-frequency signals for PLL-based circuits was proposed by Leonov (Leonov, 2008). The included papers [PI—PV] generalize this approach to the classic Costas loop with a sinusoidal VCO signal and various types of input signals. Here, we will describe the general approach to the investigation of Costas loops.

Now, consider the operation of a Costas loop before synchronization (Fig. 2), i.e., when the carrier phase $\theta^1(t)$ and the VCO phase $\theta^2(t)$ are different.

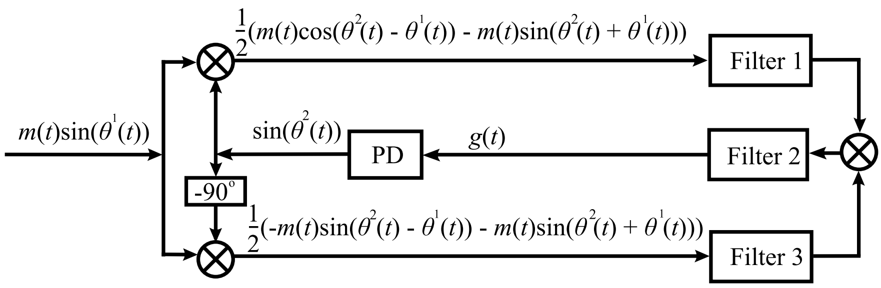


FIGURE 2 The classic Costas loop with the phase difference $\theta^2(t) - \theta^1(t)$

Let us formulate the high-frequency property of signals $f^{1,2}(t) = f^{1,2}(\theta^{1,2}(t))$ (here $f^{1,2}(\theta)$ are waveforms) in the following way: suppose that for the frequencies

$$\omega^{1,2}(t) = \dot{\theta}^{1,2}(t)$$

there exist a large number ω_{min} such that within a fixed time interval $[0, T]$ the following relation holds true:

$$\omega^{1,2}(t) \geq \omega_{min} > 0, \tag{3}$$

where T doesn't depend on ω_{min}.

The frequency difference is supposed to be uniformly bounded, i.e.,

$$\left|\omega^1(t) - \omega^2(t)\right| \leq \Delta\omega, \ \forall t \in [0, T]. \tag{4}$$

Denote $\delta = \omega_{min}^{-\frac{1}{2}}$, then

$$\begin{aligned} &|\omega^p(t) - \omega^p(\tau)| \leq \Delta\Omega, \ p = 1, 2, \\ &|t - \tau| \leq \delta, \quad \forall t, \tau \in [0, T], \end{aligned} \tag{5}$$

where $\Delta\Omega$ doesn't depend on δ.

Following the application of a Costas loop to GPS (Kaplan and Hegarty, 2006), let us consider a simplified loop shown in Fig. 3. It is the same loop as in Fig. 2, yet without Filter 1 and Filter 3, $m(t) \equiv 1$.

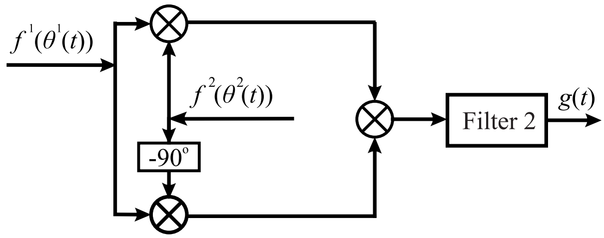


FIGURE 3 Simplified Costas loop

For the piecewise differentiable signal waveforms $f^{1,2}(\theta)$, which can be presented in the form of Fourier series

$$\begin{aligned} &f^p(\theta) = \tfrac{a_0}{2} + \sum_{i=1}^{\infty} \left(a_i^p \cos(i\theta) + b_i^p \sin(i\theta)\right), \quad \theta \geq 0 \\ &a_0^p = \frac{1}{\pi} \int_{-\pi}^{\pi} f^p(\theta) d\theta, \ a_i^p = \frac{1}{\pi} \int_{-\pi}^{\pi} f^p(\theta) \cos(i\theta) d\theta, \\ &b_i^p = \frac{1}{\pi} \int_{-\pi}^{\pi} f^p(\theta) \sin(i\theta) d\theta, \quad p = 1, 2, \end{aligned}$$

it is possible to obtain the phase detector characteristics. Let us assume, that the linear filter satisfies the relation

$$\begin{aligned} \sigma(t) = \alpha_0(t) + \int_0^t \gamma(t - \tau)\xi(\tau) \mathrm{d}\tau, \\ |\gamma(\tau) - \gamma(t)| = O(\delta), \quad |t - \tau| \leq \delta, \quad \forall \tau, t \in [0, T], \end{aligned} \tag{6}$$

where $\xi(t)$ and $\sigma(t)$ are filter's input and output respectively, $\gamma(t)$ is the impulse transient function with bounded derivative $\alpha_0(t)$ is exponentially damped function depending on the initial conditions of the filter.

Using relation (6) we get

$$\begin{aligned} g(t) = \alpha_0(t) + &\int_0^t \gamma(t - \tau) f^1\big(\theta^1(\tau)\big) f^2\big(\theta^2(\tau)\big) \\ &f^1\big(\theta^1(\tau)\big) f^2\big(\theta^2(\tau) - \frac{\pi}{2}\big) d\tau. \end{aligned} \tag{7}$$

Consider the block-scheme shown in Fig. 4. Here, the phase detector (PD)

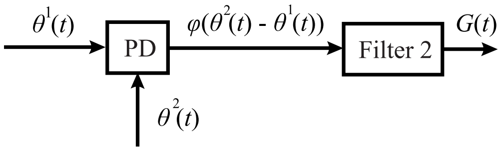


FIGURE 4 Phase detector and filter

is a nonlinear element with the output $\varphi\big(\theta^1(t) - \theta^2(t)\big)$, which represents all intermediate multipliers; $G(t)$ is the output of the filter.

Let the initial conditions of the filters in Fig. 3 and Fig. 4 be the same, then

$$G(t) = \alpha_0(t) + \int\limits_0^t \gamma(t-\tau)\varphi\big(\theta^1(\tau) - \theta^2(\tau)\big)d\tau. \tag{8}$$

Consider a 2π-periodic function

$$\begin{aligned}\varphi(\theta) = \frac{A_0^1 A_0^2}{4} + \frac{1}{2}\sum_{l=1}^{\infty}\Big((A_l^1 A_l^2 + B_l^1 B_l^2)\cos(l\theta)+ \\ (A_l^1 B_l^2 - B_l^1 A_l^2)\sin(l\theta)\Big),\end{aligned} \tag{9}$$

where A_l^p, B_l^p can be calculated from the Fourier series coefficients of $f^{1,2}(\theta)$ as follows

$$\begin{aligned}
A_l^1 &= \frac{a_0^1 a_l^1}{2} + \frac{1}{2}\sum_{m=1}^{\infty}[a_m^1(a_{m+l}^1 + a_{m-l}^1) + b_m^1(b_{m+l}^1 + b_{m-l}^1)],\\
B_l^1 &= \frac{a_0^1 b_l^1}{2} + \frac{1}{2}\sum_{m=1}^{\infty}[a_m^1(b_{m+l}^1 - b_{m-l}^1) - b_m^1(a_{m+l}^1 - a_{m-l}^1)],\\
A_l^2 &= \frac{a_0^2 \alpha_l^2}{2} + \frac{1}{2}\sum_{m=1}^{\infty}[\alpha_m^2(\alpha_{m+l}^2 + \alpha_{m-l}^2) + \beta_m^2(\beta_{m+l}^2 + \beta_{m-l}^2)],\\
B_l^2 &= \frac{a_0^2 \beta_l^2}{2} + \frac{1}{2}\sum_{m=1}^{\infty}[\alpha_m^2(\beta_{m+l}^2 - \beta_{m-l}^2 - \beta_m^2(\alpha_{m+l}^2 - \alpha_{m-l}^2)],
\end{aligned} \tag{10}$$

where

$$\alpha_k^2 = \begin{cases} a_k^2, & k = 4p, \\ b_k^2, & k = 4p+1, \\ -a_k^2, & k = 4p+2, \\ -b_k^2, & k = 4p+3, \end{cases} \beta_k^2 = \begin{cases} b_k^2, & k = 4p, \\ -a_k^2, & k = 4p+1, \\ -b_k^2, & k = 4p+2, \\ a_k^2, & k = 4p+3, \end{cases} \tag{11}$$
$$a_{-h} = a_h,\ b_{-h} = b_h,\ h < 0.$$

The following theorem allows one prove the equivalence of the models shown in Fig. 3 and Fig. 4.

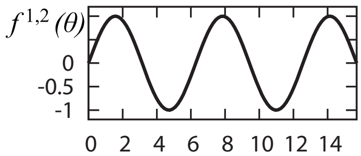
Theorem 1. If (3)–(6), are satisfied then

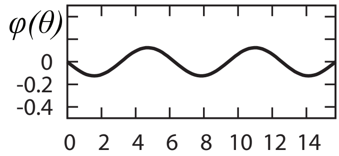
$$|g(t) - G(t)| = O(\delta), \quad \forall t \in [0, T]. \tag{12}$$

The proof of this theorem with some additional clarification can be found in the included articles [PI–PV].

Phase detector characteristics examples

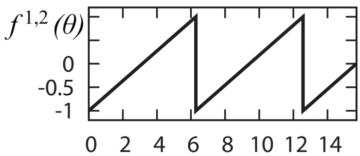
$f^{1,2}(\theta) = \sin(\theta)$, $\varphi(\theta) = -\frac{1}{8}\sin(2\theta)$

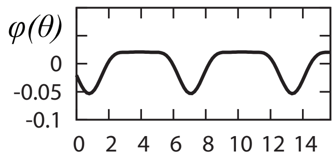




$$f^{1,2}(\theta) = -\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\big(n\theta\big),$$

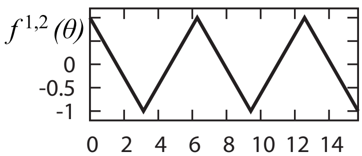
$$\varphi(\theta)=\frac{-1}{72} + \frac{1}{2}\sum_{l=1}^{\infty}\begin{cases} \frac{16}{\pi^4 l^4}\cos(l\theta), \quad l = 4p, \ p \in \mathbb{N} \\ \frac{-4(\pi l-2)}{\pi^4 l^4}\cos(l\theta)\frac{-4(\pi l-2)}{\pi^4 l^4}\sin(l\theta), \quad l = 4p+1, \ p \in \mathbb{N}_0 \\ \frac{-8}{\pi^3 l^3}\sin(l\theta), \quad l = 4p+2, \ p \in \mathbb{N}_0 \\ \frac{4(\pi l+2)}{\pi^4 l^4}\cos(l\theta)\frac{-4(\pi l+2)}{\pi^4 l^4}\sin(l\theta), \quad l = 4p+3, \ p \in \mathbb{N}_0 \end{cases}$$

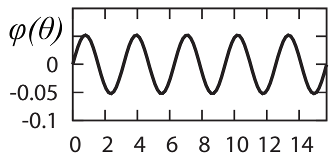




$$f^{1,2}(\theta) = \frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos\big((2n-1)\theta\big)$$

$$\varphi(\theta) = \frac{512}{\pi^5}\sum_{l=1}^{\infty}\frac{1}{(4l+2)^5}\sin((4l+2)\theta)$$

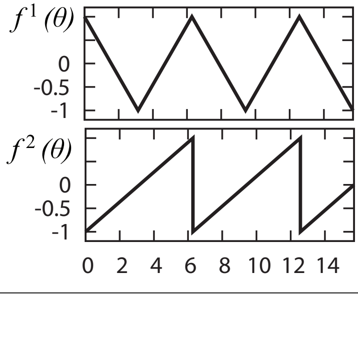


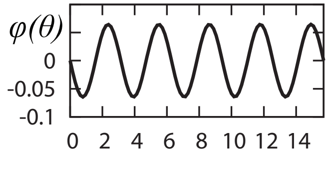


$$f^{1}(\theta) = \frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos\big((2n-1)\theta\big)$$

$$f^{2}(\theta) = -\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\big(n\theta\big)$$

$$\varphi(\theta) = \frac{-1}{72}+\frac{8}{\pi^2}\sum_{l=1}^{\infty}\begin{cases} \frac{4}{\pi^2(l+4)^4}\cos((l+4)\theta), \ l = 4p, \\ -\frac{2}{\pi l^3}\sin(l\theta), \ l = 4p+2, \\ 0, \quad l = 2p+1, \ p \in \mathbb{N}_0 \end{cases}$$





2.2 QPSK Costas loop

The following description of the QPSK Costas loop follows article [PV]. Below, we provide a rigorous mathematical description of the QPSK Costas loop and formulate the corresponding theorem.

Consider the QPSK Costas loop shown in Fig. 5. The input signal has the

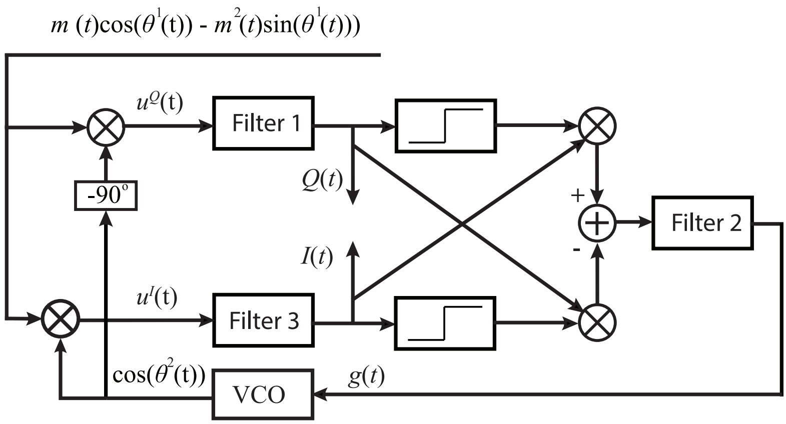


FIGURE 5 The QPSK Costas loop

following form

$$m^1(t)\cos(\theta^1(t)) - m^2(\sin(\theta^1(t))),$$

where $m^{1,2}(t) = \pm 1$ is the transmitted data, $\sin(\theta^1(t))$ and $\cos(\theta^1(t))$ are carriers. Consider a sinusoidal output of VCO $\cos(\theta^2(t))$. The input signal and VCO output are high-frequency signals, i.e., for $\theta^1(t)$ and $\theta^2(t)$ conditions (3)–(5) are satisfied.

On the lower branch (I), after the multiplying input signal by the VCO signal, we get

$$u^I(t) = \big(m^1(t)\cos(\theta^1(t)) - m^2(\sin(\theta^1(t)))\big)\cos(\theta^2(t)),$$

Then, a low-pass filter (Filter 3) forms the signal

$$I(t) = \int_0^t h(t-\tau)u^I(\tau)d\tau,$$

where $h(t-\tau)$ is an impulse transient function. The signal $I(t)$ allows one to obtain one of the carriers of the input signal.

Similarly, on the upper branch (Q) the product of the input signal and the VCO signal, shifted by -90^o, forms the signal

$$u^Q(t) = \big(m^1(t)\cos(\theta^1(t)) - m^2(\sin(\theta^1(t)))\big)\sin(\theta^2(t)).$$

After the filtration by a low-pass filter (Filter 1) we get

$$Q(t) = \int_0^t h(t-\tau)u^Q(\tau)d\tau.$$

This signal allows one to obtain the second carrier of the input signal. After the filtration, both signals, $I(t)$ and $Q(t)$, pass through the limiters. The outputs of the limiters are equal to $\text{sign}(I(t))$ and $\text{sign}(Q(t))$, respectively. Then, these signals are multiplied as shown in Fig. 5. The resulting difference, after the filtration by Filter 2, forms the control signal $g(t)$. This signal is used as the input of the VCO for frequency and phase corrections. Similarly to the classic Costas loop, Filter 2 satisfies conditions (6).

Based on the applications of the QPSK Costas loop to GPS (Kaplan and Hegarty, 2006), we may consider a simplified loop (see Fig. 6). Here $m^{1,2}(t) \equiv 1$.

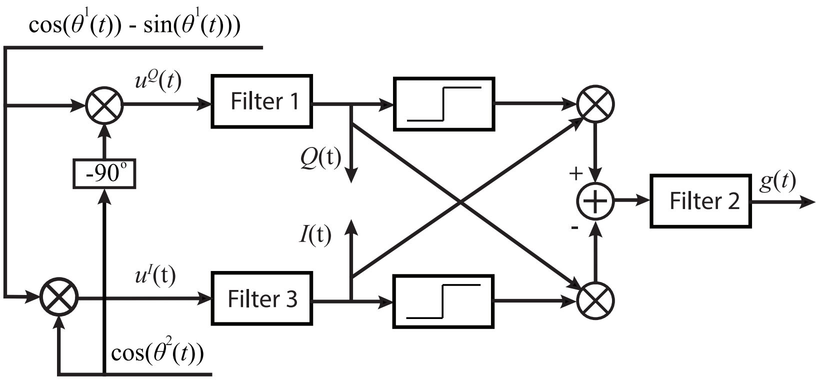


FIGURE 6 A simplified QPSK Costas loop

Suppose that Filter 1 and Filter 3 satisfy the conditions

$$\begin{aligned} &\int_0^t h(t-\tau)\sin(\omega\tau)d\tau = O(\frac{1}{\omega}),\ \forall \omega > \omega_{min}, \\ &\int_0^t h(t-\tau)\sin(\omega\tau)d\tau = \sin(\omega\tau) + O(\frac{1}{\omega}),\ \forall \omega < \Delta\omega. \end{aligned} \tag{13}$$

Consider the block-scheme shown in Fig. 4, where Filter 2 is the same filter as the one in Fig. 6

Consider a 2π-periodic function $\varphi(\theta)$

$$\begin{aligned} \varphi(\theta) =& 0.5\sqrt{2}\sin(\theta(t))\,\text{sign}(\sin(\theta(t)))- \\ &-\sin(\theta(t))\,\text{sign}(\sin(\theta(t)). \end{aligned} \tag{14}$$

The following theorem allows one to justify the transition from the block-scheme shown in Fig. 6 to that of Fig. 4.

Theorem 2. If conditions (3)–(6) and (13) are satisfied, then

$$|g(t) - G(t)| = O(\delta), \quad \forall t \in [0, T]. \tag{15}$$

This result is discussed in details in (Yuldashev, 2013a).

2.3 Differential equations of Costas loops

A physical model of the classic Costas loop can be described by the following system of differential equations

$$\begin{aligned} \dot{x} &= Ax + bf^1(\theta^1(t))f^2(\theta^2(t))f^1(\theta^1(t))f^2(\theta^2(t) - \frac{\pi}{2}), \\ \dot{\theta}^2 &= \omega^2_{free} + Lc^*x, \\ \dot{\theta}^1 &\equiv \omega^1. \end{aligned} \tag{16}$$

Here A is the constant matrix of the filter, $x(t)$ represents the state of the filter, b and c are constant vectors — parameters of the filter, L is a constant, which defines feedback strength of the system, ω^2_{free} is the VCO self (free) frequency, * is the transpose operator.

The QPSK Costas loop can be described by the following differential equations:

$$\begin{aligned} \dot{x_1} &= A_1x_1 + b_1(\cos(\theta^2)(\cos(\theta^1) - \sin(\theta^1))), \\ \dot{x_2} &= A_2x_2 + b_2(\text{sign}(c_1^*x_1)(c_3^*x_3) - \text{sign}(c_3^*x_3)(c_1^*x_1)), \\ \dot{x_3} &= A_3x_3 + b_3(\sin(\theta^2)(\cos(\theta^1) - \sin(\theta^1))), \\ \dot{\theta}^1 &\equiv \omega^1, \\ \dot{\theta}^2 &= \omega^2_{free} + Lc_2^*x_2, \end{aligned} \tag{17}$$

where $A_{1,2,3}$, $b_{1,2,3}$, $c_{1,2,3}$ are parameters of the filter and $x_{1,2,3}(t)$ is the state of the filter.

Using the phase-detector characteristics, it is possible to derive differential equations of Costas loops in the phase space as follows

$$\begin{aligned} \dot{x} &= Ax + b\varphi(\theta_\Delta), \\ \dot{\theta}_\Delta &= \omega^2_{free} - \omega^1 + Lc^*x, \\ \theta_\Delta &= \theta^2 - \theta^1. \end{aligned} \tag{18}$$

Here $\varphi(\theta)$ is the phase detector characteristics, which depends on the signal waveforms.

The averaging methods allow one to justify that the solutions of differential equations in the phase space are close to the solutions in the signal/time space.

2.4 Numerical simulation

The numerical simulations of the considered Costas loops with different sets of parameters in the signal space and the phase space confirms the theoretical results. In Fig. 7 one can see some results of the simulation of the classic Costas loop and that of the QPSK Costas loop. It should be noted, that the numerical simulation in the phase space is more than hundred-fold times efficient than the simulation in the signal space.

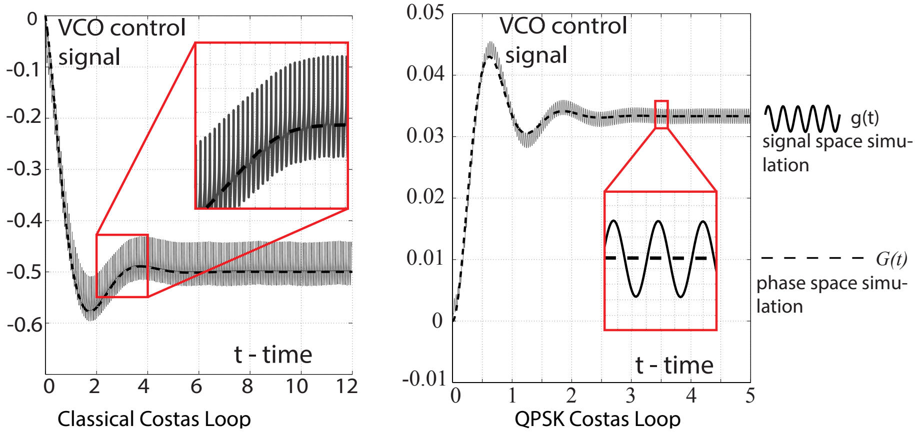


FIGURE 7 Costas loop simulation

The efficiency of the proposed method is confirmed by the numerical simulation of the Costas loop for high-frequency signals. In Fig. 8, an example of modeling the classic Costas loop with 1Ghz signals is shown. Here, in the course of 10 seconds of computing in the signal space (where the process of computation is very slow), only 2.5×10^{-7} seconds of the transient processes have been modeled. Therefore, the numerical simulation of the full transient process in the signal space is almost impossible for high-frequency signals. At the same time, the full simulation time of 20 seconds of the transient processes in the phase space took less than a second.

The derived method can be adapted for the numerical simulation of the digital Costas loops. An example of such simulation is shown in Fig. 9.

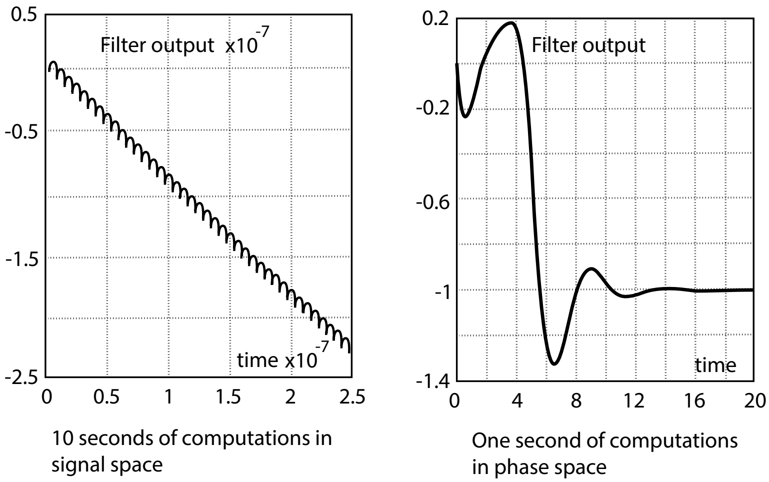


FIGURE 8 Comparison of the effectiveness of simulation in signal space and phase space

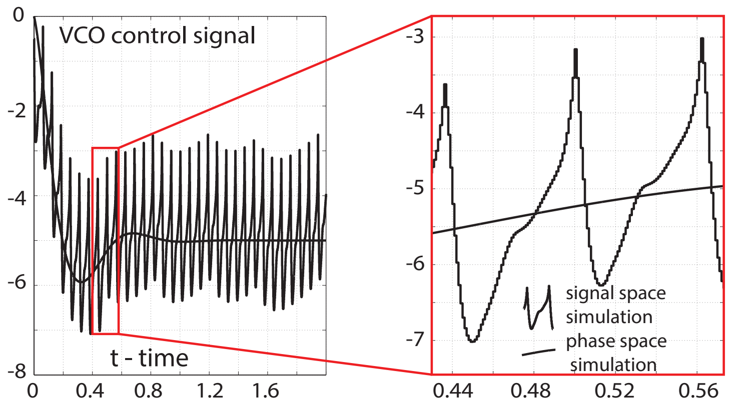


FIGURE 9 Simulation of digital Costas loop

REFERENCES

Abe, T., Yuan, Y., Ishikuro, H., and Kuroda, T. (2012). A 2gb/s 150mw uwb direct-conversion coherent transceiver with iq-switching carrier recovery scheme. In *Solid-State Circuits Conference Digest of Technical Papers (ISSCC), 2012 IEEE International*, 442–444.

Abramovitch, D. (2002). Phase-locked loops: A control centric tutorial. In *Proceedings of the American Control Conference*, volume 1, 1–15.

Abramovitch, D. (2008a). Efficient and flexible simulation of phase locked loops, part I: simulator design. In *American Control Conference*, 4672–4677. Seattle, WA.

Abramovitch, D. (2008b). Efficient and flexible simulation of phase locked loops, part II: post processing and a design example. In *American Control Conference*, 4678–4683. Seattle, WA.

An'an, Z. and Du Yong, H.F. (2006). Design and implementation of costas loop on fpga platform. *Electronic Engineer*, 1, 18–20.

Andrievsky, B.R., Kuznetsov, N.V., Leonov, G.A., and Pogromsky, A.Y. (2012). Convergence based anti-windup design method and its application to flight control. 212–218 (art. no. 6459667). doi:10.1109/ICUMT.2012.6459667.

Banerjee, T. and Sarkar, B. (2008a). Chaos and bifurcation in a third-order digital phase-locked loop. *International Journal of Electronics and Communications*, (62), 86–91.

Banerjee, T. and Sarkar, B.C. (2008b). Chaos and bifurcation in a third-order digital phase-locked loop. *International Journal of Electronics and Communications*, (62), 86–91.

Beier, W. (1987). Receiver for bandspread signals. US Patent 4,672,629.

Benarjee, D. (2006). *PLL Perforfmance, Simulation, and Design*. Dog Ear Publishing.

Best, R.E. (2007). *Phase-Lock Loops: Design, Simulation and Application*. McGraw-Hill.

Bouaricha, A. et al. (2012). Hybrid time and frequency solution for PLL sub-block simulation. US Patent 8,209,154.

Braasch, M.S. and Van Dierendonck, A. (1999). Gps receiver architectures and measurements. *Proceedings of the IEEE*, 87(1), 48–64.

Bragin, V.O., Kuznetsov, N.V., and Leonov, G.A. (2010). Algorithm for counterexamples construction for Aizerman's and Kalman's conjectures. *IFAC Proceedings Volumes (IFAC-PapersOnline)*, 4(1), 24–28. doi:10.3182/20100826-3-TR-4016.00008.

Bragin, V.O., Vagaitsev, V.I., Kuznetsov, N.V., and Leonov, G.A. (2011). Algorithms for finding hidden oscillations in nonlinear systems. The Aizerman and Kalman conjectures and Chua's circuits. *Journal of Computer and Systems Sciences International*, 50(4), 511–543. doi:10.1134/S106423071104006X.

Chang, G. and Chen, C. (2008). A comparative study of voltage flicker envelope estimation methods. In *Power and Energy Society General Meeting - Conversion and Delivery of Electrical Energy in the 21st Century*, 1–6.

Chen, R., Guan, J.x., and Zhang, X.m. (2010). Design and implementation of digital costas-loop. *Radio Engineering*, 3, 010.

Chicone, C. and Heitzman, M.T. (2013). Phase-locked loops, demodulation, and averaging approximation time-scale extensions. *SIAM Journal on Applied Dynamical Systems*, 12(2), 674–721.

Cho, P.S. (2006). Optical phase-locked loop performance in homodyne detection using pulsed and cw lo. In *Optical Amplifiers and Their Applications/Coherent Optical Technologies and Applications*, JWB24. Optical Society of America.

Costas, J.P. (1962). Receiver for communication system. US Patent 3,047,659.

Costas, J. (1956). Synchoronous communications. In *Proc. IRE*, volume 44, 1713–1718.

Couch, L. (2007). *Digital and Analog Communication Systems*. Pearson/Prentice Hall.

Demir, A., Mehrotra, A., and Roychowdhury, J. (2000a). Phase noise in oscillators: a unifying theory and numerical methods for characterization. *IEEE Transactions on Circuits and Systems I*, 47, 655–674.

Demir, A., Mehrotra, A., and Roychowdhury, J. (2000b). Phase noise in oscillators: a unifying theory and numerical methods for characterization. *IEEE Transactions on Circuits and Systems I*, 47, 655–674.

Djordjevic, I.B. and Stefanovic, M.C. (1999). Performance of optical heterodyne psk systems with Costas loop in multichannel environment for nonlinear second-order PLL model. *J. Lightwave Technol.*, 17(12), 2470.

Djordjevic, I.B., Stefanovic, M.C., Ilic, S.S., and Djordjevic, G.T. (1998). An example of a hybrid system: Coherent optical system with Costas loop in receiver-system for transmission in baseband. *J. Lightwave Technol.*, 16(2), 177.

Egan, W.F. (2000). *Frequency synthesis by phase lock*. Wiley New York.

Emura, T. (1982). A study of a servomechanism for nc machines using 90 degrees phase difference method. *Prog. Rep. of JSPE*, 419–421.

Feely, O. (2007). Nonlinear dynamics of discrete-time circuits: A survey. *International Journal of Circuit Theory and Applications*, (35), 515–531.

Feely, O., Curran, P.F., and Bi, C. (2012). Dynamics of charge-pump phase-locked loops. *International Journal of Circuit Theory and Applications*. doi:10.1002/cta.

Fines, P. and Aghvami, A. (1991). Fully digital m-ary psk and m-ary qam demodulators for land mobile satellite communications. *IEEE Electronics and Communication Engineering Journal*, 3(6), 291–298.

Fiocchi, C., Maloberti, F., and Torelli, G. (1992). A sigma-delta based PLL for non-sinusoidal waveforms. In *ISCAS' 92, IEEE International Symposium on*, volume 6.

Gao, W. and Feher, K. (1996). All-digital reverse modulation architecture based carrier recovery implementation for gmsk and compatible fqpsk. *Broadcasting, IEEE Transactions on*, 42(1), 55–62.

Gardner, F. (1966). *Phase–lock techniques*. John Wiley, New York.

Gardner, F. (1993). Interpolation in digital modems - part I: Fundamentals. *IEEE Electronics and Communication Engineering Journal*, 41(3), 501–507.

Gardner, F., Erup, L., and Harris, R. (1993). Interpolation in digital modems - part II: Implementation and performance. *IEEE Electronics and Communication Engineering Journal*, 41(6), 998–1008.

Hayami, Y., Imai, F., and Iwashita, K. (2008). Linewidth investigation for costas loop phase-diversity homodyne detection in digital coherent detection system. In *Asia Optical Fiber Communication and Optoelectronic Exposition and Conference*, SaK20. Optical Society of America.

Hegarty, C.J. (2012). Gnss signals - an overview. In *Frequency Control Symposium (FCS), 2012 IEEE International*, 1–7.

Henning, F.H. (1981). *Nonsinusoidal Waves for Radar and Radio Communication*. Academic Pr, first edition.

Hodgkinson, T. (1986). Costas loop analysis for coherent optical receivers. *Electronics letters*, 22(7), 394–396.

Hu, Y. and Sawan, M. (2005). A fully integrated low-power bpsk demodulator for implantable medical devices. *Circuits and Systems I: Regular Papers, IEEE Transactions on*, 52(12), 2552–2562.

Humphreys, T.E., Psiaki, M.L., Ledvina, B.M., and Kintner Jr, P. (2005). Gps carrier tracking loop performance in the presence of ionospheric scintillations. *Proceedings of ION GNSS 2005*, 13–16.

Jasper, S.C. (1987). Method of doppler searching in a digital gps receiver. US Patent 4,701,934.

Kaplan, E. and Hegarty, C. (2006). *Understanding GPS: Principles and Applications*. Artech House.

Kim, H., Kang, S., Chang, J.H., Choi, J.H., Chung, H., Heo, J., Bae, J.D., Choo, W., and Park, B.h. (2010). A multi-standard multi-band tuner for mobile TV SoC with GSM interoperability. In *Radio Frequency Integrated Circuits Symposium (RFIC), 2010*, 189–192. IEEE.

Kiseleva, M.A., Kuznetsov, N.V., Leonov, G.A., and Neittaanmäki, P. (2012). Drilling systems failures and hidden oscillations. In *IEEE 4th International Conference on Nonlinear Science and Complexity, NSC 2012 - Proceedings*, 109–112. doi: 10.1109/NSC.2012.6304736.

Kobayashi, K., Matsumoto, Y., Seki, K., and Kato, S. (1992). A full digital modem for offset type modulation schemes. In *Personal, Indoor and Mobile Radio Communications, 1992. Proceedings, PIMRC'92., Third IEEE International Symposium on*, 596–599.

Kroupa, V. (2003). *Phase Lock Loops and Frequency Synthesis*. John Wiley & Sons.

Krylov, N. and Bogolyubov, N. (1947). *Introduction to non-linear mechanics*. Princeton Univ. Press, Princeton.

Kudrewicz, J. and Wasowicz, S. (2007). *Equations of phase-locked loop. Dynamics on circle, torus and cylinder*, volume 59 of *A*. World Scientific.

Kuznetsov, N., Kuznetsova, O., Leonov, G., and Vagaitsev, V. (2013a). *Informatics in Control, Automation and Robotics, Lecture Notes in Electrical Engineering, Volume 174, Part 4*, chapter Analytical-numerical localization of hidden attractor in electrical Chua's circuit, 149–158. Springer. doi:10.1007/978-3-642-31353-0_11.

Kuznetsov, N.V. (2008). *Stability and Oscillations of Dynamical Systems: Theory and Applications*. Jyvaskyla University Printing House.

Kuznetsov, N.V., Kuznetsova, O.A., and Leonov, G.A. (2013b). Visualization of four normal size limit cycles in two-dimensional polynomial quadratic system. *Differential equations and dynamical systems*, 21(1-2), 29–34. doi:10.1007/s12591-012-0118-6.

Kuznetsov, N.V., Kuznetsova, O.A., Leonov, G.A., and Vagaytsev, V.I. (2011a). Hidden attractor in Chua's circuits. *ICINCO 2011 - Proceedings of the 8th International Conference on Informatics in Control, Automation and Robotics*, 1, 279–283. doi:10.5220/0003530702790283.

Kuznetsov, N.V. and Leonov, G.A. (2001). Counterexample of Perron in the discrete case. *Izv. RAEN, Diff. Uravn.*, 5, 71.

Kuznetsov, N.V. and Leonov, G.A. (2005). On stability by the first approximation for discrete systems. *2005 International Conference on Physics and Control, PhysCon 2005*, Proceedings Volume 2005, 596–599. doi:10.1109/PHYCON.2005.1514053.

Kuznetsov, N.V. and Leonov, G.A. (2008). Lyapunov quantities, limit cycles and strange behavior of trajectories in two-dimensional quadratic systems. *Journal of Vibroengineering*, 10(4), 460–467.

Kuznetsov, N.V., Leonov, G.A., Neittaanmäki, P., Seledzhi, S.M., Yuldashev, M.V., and Yuldashev, R.V. (2010a). Nonlinear analysis of phase-locked loop. *IFAC Proceedings Volumes (IFAC-PapersOnline)*, 4(1), 34–38. doi:10.3182/20100826-3-TR-4016.00010.

Kuznetsov, N.V., Leonov, G.A., and Seledzhi, S.M. (2009a). Nonlinear analysis of the Costas loop and phase-locked loop with squarer. In *Proceedings of the IASTED International Conference on Signal and Image Processing, SIP 2009*, 1–7.

Kuznetsov, N.V., Leonov, G.A., and Seledzhi, S.M. (2011b). Hidden oscillations in nonlinear control systems. *IFAC Proceedings Volumes (IFAC-PapersOnline)*, 18(1), 2506–2510. doi:10.3182/20110828-6-IT-1002.03316.

Kuznetsov, N.V., Leonov, G.A., Seledzhi, S.M., and Neittaanmäki, P. (2009b). Analysis and design of computer architecture circuits with controllable delay line. *ICINCO 2009 - 6th International Conference on Informatics in Control, Automation and Robotics, Proceedings*, 3 SPSMC, 221–224. doi:10.5220/0002205002210224.

Kuznetsov, N.V., Leonov, G.A., and Seledzhi, S.S. (2008). Phase locked loops design and analysis. In *ICINCO 2008 - 5th International Conference on Informatics in Control, Automation and Robotics, Proceedings*, volume SPSMC, 114–118. doi:10.5220/0001485401140118.

Kuznetsov, N.V., Leonov, G.A., and Vagaitsev, V.I. (2010b). Analytical-numerical method for attractor localization of generalized Chua's system. *IFAC Proceedings Volumes (IFAC-PapersOnline)*, 4(1), 29–33. doi:10.3182/20100826-3-TR-4016.00009.

Kuznetsov, N.V., Vagaytsev, V.I., Leonov, G.A., and Seledzhi, S.M. (2011c). Localization of hidden attractors in smooth Chua's systems. *International Conference on Applied and Computational Mathematics*, 26–33.

Leonov, G.A. (2006). Phase-locked loops. theory and application. *Automation and Remote Control*, 10, 47–55.

Leonov, G.A. (2008). Computation of phase detector characteristics in phase-locked loops for clock synchronization. *Doklady Mathematics*, 78(1), 643–645.

Leonov, G.A. (2010). Effective methods for periodic oscillations search in dynamical systems. *App. math. & mech.*, 74(1), 24–50.

Leonov, G.A., Andrievskii, B.R., Kuznetsov, N.V., and Pogromskii, A.Y. (2012a). Aircraft control with anti-windup compensation. *Differential equations*, 48(13), 1700–1720. doi:10.1134/S001226611213.

Leonov, G.A., Bragin, V.O., and Kuznetsov, N.V. (2010a). Algorithm for constructing counterexamples to the Kalman problem. *Doklady Mathematics*, 82(1), 540–542. doi:10.1134/S1064562410040101.

Leonov, G.A., Bragin, V.O., and Kuznetsov, N.V. (2010b). On problems of Aizerman and Kalman. *Vestnik St. Petersburg University. Mathematics*, 43(3), 148–162. doi:10.3103/S1063454110030052.

Leonov, G.A. and Kuznetsov, G.V. (2013a). Hidden oscillations in drilling systems: torsional vibrations. *Journal of Applied Nonlinear Dynamics*, 2(1), 83–94. doi:10.5890/JAND.2012.09.006.

Leonov, G.A. and Kuznetsov, N.V. (2007). Time-varying linearization and the Perron effects. *International Journal of Bifurcation and Chaos*, 17(4), 1079–1107. doi:10.1142/S0218127407017732.

Leonov, G.A. and Kuznetsov, N.V. (2010). Limit cycles of quadratic systems with a perturbed weak focus of order 3 and a saddle equilibrium at infinity. *Doklady Mathematics*, 82(2), 693–696. doi:10.1134/S1064562410050042.

Leonov, G.A. and Kuznetsov, N.V. (2011). Analytical-numerical methods for investigation of hidden oscillations in nonlinear control systems. *IFAC Proceedings Volumes (IFAC-PapersOnline)*, 18(1), 2494–2505. doi:10.3182/20110828-6-IT-1002.03315.

Leonov, G.A. and Kuznetsov, N.V. (2012). IWCFTA2012 keynote speech I - Hidden attractors in dynamical systems: From hidden oscillation in Hilbert-Kolmogorov, Aizerman and Kalman problems to hidden chaotic attractor in Chua circuits. In *Chaos-Fractals Theories and Applications (IWCFTA), 2012 Fifth International Workshop on*, XV–XVII. doi:10.1109/IWCFTA.2012.8.

Leonov, G.A. and Kuznetsov, N.V. (2013b). Hidden attractors in dynamical systems. From hidden oscillations in Hilbert-Kolmogorov, Aizerman, and Kalman problems to hidden chaotic attractors in Chua circuits. *International Journal of Bifurcation and Chaos*, 23(1). doi:10.1142/S0218127413300024. art. no. 1330002.

Leonov, G.A. and Kuznetsov, N.V. (2013c). Hidden oscillations in dynamical systems. From hidden oscillation in 16th Hilbert, Aizerman and Kalman problems to hidden chaotic attractor in Chua circuits. In *Chaos-Fractals Theories and Applications (IWCFTA), 2012 Fifth International Workshop on*, XV–XVII. doi:10.1109/IWCFTA.2012.8.

Leonov, G.A. and Kuznetsov, N.V. (2013d). *Numerical Methods for Differential Equations, Optimization, and Technological Problems, Computational Methods in Applied Sciences, Volume 27, Part 1*, chapter Analytical-numerical methods for hidden attractors' localization: the 16th Hilbert problem, Aizerman and Kalman conjectures, and Chua circuits, 41–64. Springer. doi:10.1007/978-94-007-5288-7_3.

Leonov, G.A., Kuznetsov, N.V., and Kudryashova, E.V. (2011a). A direct method for calculating Lyapunov quantities of two-dimensional dynamical systems. *Proceedings of the Steklov Institute of Mathematics*, 272 (Suppl. 1), S119–S127. doi: 10.1134/S008154381102009X.

Leonov, G.A., Kuznetsov, N.V., Kuznetsova, O.A., Seledzhi, S.M., and Vagaitsev, V.I. (2011b). Hidden oscillations in dynamical systems. *Transaction on Systems and Control*, 6(2), 54–67.

Leonov, G.A., Kuznetsov, N.V., and Seledzhi, S.M. (2006). Analysis of phase-locked systems with discontinuous characteristics. *IFAC Proceedings Volumes (IFAC-PapersOnline)*, 1, 107–112. doi:10.3182/20060628-3-FR-3903.00021.

Leonov, G.A., Kuznetsov, N.V., and Seledzhi, S.M. (2009). *Automation control - Theory and Practice*, chapter Nonlinear Analysis and Design of Phase-Locked Loops, 89–114. In-Tech. doi:10.5772/7900.

Leonov, G.A., Kuznetsov, N.V., and Seledzhi, S.M. (2011c). Hidden oscillations in dynamical systems. *Recent researches in System Science*, 292–297.

Leonov, G.A., Kuznetsov, N.V., and Vagaitsev, V.I. (2011d). Localization of hidden Chua's attractors. *Physics Letters A*, 375(23), 2230–2233. doi:10.1016/j.physleta.2011.04.037.

Leonov, G.A., Kuznetsov, N.V., and Vagaitsev, V.I. (2012b). Hidden attractor in smooth Chua systems. *Physica D: Nonlinear Phenomena*, 241(18), 1482–1486. doi: 10.1016/j.physd.2012.05.016.

Leonov, G.A., Vagaitsev, V.I., and Kuznetsov, N.V. (2010c). Algorithm for localizing Chua attractors based on the harmonic linearization method. *Doklady Mathematics*, 82(1), 693–696. doi:10.1134/S1064562410040411.

Leonov, G. and Seledzhi, S. (2002). *The Phase-Locked Loop for Array Processors*. Nevskii dialect, St.Petersburg [in Russian].

Lin, V., Ghoneim, A., and Dafesh, P. (2004). Implementation of reconfigurable software radio for multiple wireless standards. In *Aerospace Conference, 2004. Proceedings. 2004 IEEE*, volume 2, 1392–1397.

Lindsey, W. (1972). *Synchronization systems in communication and control*. Prentice-Hall, New Jersey.

Lindsey, W. and Simon, M. (1973). *Telecommunication Systems Engineering*. Prentice Hall, NJ.

Luo, Z. and Sonkusale, S. (2008). A novel bpsk demodulator for biological implants. *Circuits and Systems I: Regular Papers, IEEE Transactions on*, 55(6), 1478–1484.

Malyon, D. (1984). Digital fibre transmission using optical homodyne detection. *Electronics Letters*, 20(7), 281–283.

Manassewitsch, V. (2005). *Frequency synthesizers: theory and design*. Wiley.

Margaris, W. (2004). *Theory of the Non-Linear Analog Phase Locked Loop*. Springer Verlag, New Jersey.

Mileant, A. and Hinedi, S. (1994). Overview of arraying techniques for deep space communications. *Communications, IEEE Transactions on*, 42(234), 1856–1865.

Misra, R. and Palod, S. (2011). Code and carrier tracking loops for gps c/a code. *Int. J. Pure Appl. Sci. Technol*, 6(1), 1–20.

Mitropolsky, Y. and Bogolubov, N. (1961). *Asymptotic Methods in the Theory of Non-Linear Oscillations*. Gordon and Breach, New York.

Miyazaki, T., Ryu, S., Namihira, Y., and Wakabayashi, H. (1991). Optical costas loop experiment using a novel optical 90 hybrid module and a semiconductor-laser-amplifier external phase adjuster. In *Optical Fiber Communication*, WH6. Optical Society of America.

Nowsheen, N., Benson, C., and Frater, M. (2010). A high data-rate, software-defined underwater acoustic modem. In *OCEANS 2010*, 1–5. IEEE.

Proakis, J.G. and Salehi, M. (2007). *Digital communications*. McGraw-Hill Higher Education, 5th edition.

Razavi, B. (2003). *Phase-Locking in High-Performance Systems: From Devices to Architectures*.

Roberts, K., O'Sullivan, M., Wu, K.T., Sun, H., Awadalla, A., Krause, D.J., and Laperle, C. (2009). Performance of dual-polarization QPSK for optical transport systems. *Journal of lightwave technology*, 27(16), 3546–3559.

Sarkar, A. and Sengupta, S. (2010). Second-degree digital differentiator-based power system frequency estimation under non-sinusoidal conditions. *IET Sci. Meas. Technol.*, 4(2), 105–114.

Shah, S. and Sinha, V. (2009). Gmsk demodulator using costas loop for software-defined radio. In *Advanced Computer Control, 2009. ICACC'09. International Conference on*, 757–761. IEEE.

Shu, K. and Sanchez-Sinencio, E. (2005). *CMOS PLL synthesizers: analysis and design*. Springer.

Stephens, R.D. (2001). *Phase-Locked Loops for Wireless Communications: Digital, Analog and Optical Implementations*. Springer.

Suarez, A. and Quere, R. (2003). *Stability Analysis of Nonlinear Microwave Circuits*. Artech House, New Jersey.

Suarez, A., Fernandez, E., Ramirez, F., and Sancho, S. (2012). Stability and bifurcation analysis of self-oscillating quasi-periodic regimes. *IEEE transactions on microwave theory and techniques*, 60(3), 528–541.

Sutterlin, P. and Downey, W. (1999). A power line communication tutorial - challenges and technologies. In *Technical Report*. Echelon Corporation.

Tanaka, K., Muto, T., Hori, K., Wakamori, M., Teranishi, K., Takahashi, H., Sawada, M., and Ronning, M. (2001). A high performance gps solution for mobile use. In *Proceedings of the 15th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GPS 2002)*, 1648–1655.

Tang, X.M., Xu, P.C., and Wang, F.X. (2010). Performance comparison of phase detector in navigation receiver's tracking loop. *Journal of National University of Defense Technology*, 32(2), 85–90.

Tomasi, W. (2001). *Electronic communications systems: fundamentals through advanced*. Pearson/Prentice Hall.

Tomkins, A., Aroca, R.A., Yamamoto, T., Nicolson, S.T., Voinigescu, S., et al. (2009). A zero-IF 60 GHz 65 nm CMOS transceiver with direct BPSK modulation demonstrating up to 6 Gb/s data rates over a 2 m wireless link. *Solid-State Circuits, IEEE Journal of*, 44(8), 2085–2099.

Tretter, S.A. (2007). *Communication System Design Using DSP Algorithms with Laboratory Experiments for the TMS320C6713TM DSK*. Springer.

Troedsson, N. (2009). Method and simulator for generating phase noise in system with phase-locked loop. US Patent App. 12/371,828.

Viterbi, A. (1966). *Principles of coherent communications*. McGraw-Hill, New York.

Viterbi, A. (1983). Nonlinear estimation of psk-modulated carrier phase with application to burst digital transmission. *Information Theory, IEEE Transactions on*, 29(4), 543–551.

Wang, L. and Emura, T. (1998). A high-precision positioning servo-controller using non-sinusoidal two-phase type PLL. In *UK Mechatronics Forum International Conference*, 103–108. Elsevier Science Ltd.

Wang, L. and Emura, T. (2001). Servomechanism using traction drive. *JSME International Journal Series C*, 44(1), 171–179.

Wang, Y. and Leeb, W.R. (1987). A 90 optical fiber hybrid for optimal signal power utilization. *Appl. Opt.*, 26(19), 4181–4184. doi:10.1364/AO.26.004181.

Xanthopoulos, T., Bailey, D., Gangwar, A., Gowan, M., Jain, A., and Prewitt, B. (2001). The design and analysis of the clock distribution network for a 1.2 GHz Alpha microprocessor. In *Solid-State Circuits Conference, 2001. Digest of Technical Papers. ISSCC. IEEE International*, 402–403.

Xu, W., Luo, Z., and Sonkusale, S. (2009). Fully digital bpsk demodulator and multilevel lsk back telemetry for biomedical implant transceivers. *Circuits and Systems II: Express Briefs, IEEE Transactions on*, 56(9), 714–718.

Young, I., Greason, J., and Wong, K. (1992). A PLL clock generator with 5 to 110 MHz of lock range for microprocessors. *Solid-State Circuits, IEEE Journal of*, 27(11), 1599–1607.

Young, P. (2004). *Electronic communication techniques*. Pearson/Prentice Hall.

Yu, G., Xie, X., Zhao, W., Wang, W., and Yan, S. (2011). Impact of phase noise on coherent bpsk homodyne systems in long-haul optical fiber communications. In *Photonics and Optoelectronics Meetings 2011*, 83310R–83310R. International Society for Optics and Photonics.

Yuldashev, M.V. (2012). Nonlinear analysis of costas loop. M.Sc. thesis.

Yuldashev, M.V. (2013a). Nelinejnye matematicheskie modeli skhem kostasa. Candidate of Science dissertation [in Russian].

Yuldashev, M.V. (2013b). Nonlinear analysis of costas loops. PhD thesis [in preparation].